Mathematical Modelling for Thermal Expansion of [illegible] during Roasting

A mathematical model, based on theory of thermo-elasticity, heat and mass balances and perfect gas law, has been developed for predicting volumetric thermal expansion of gorgon nut during roasting. Change in volume of nut at 2.5 min roasting time, 213°C nut temperature, and 335°C roasting pan temperature, is found to be 37.2%. The accuracy of the model's results has been examined by comparing the predicted and measured values of changes in volume of the nut during roasting. The predicted values varied from measured one by less than 10% after 2 min of roasting and thus, the model can be used for practical purposes.

Keywords : Gorgon nut, Roasting, Kernel, Popping.

INTRODUCTION

Gorgon nut (*Euryale ferox*), seed of an aquatic herb, is grown in shallow water in eastern and north-eastern states of India. Its wild populations are also available in Bangladesh, China, Japan and North America. It is characterized by its black colour, round shape (diameter ranging from 4.5 mm to 15 mm) and hard seed coat[1]. The nut is popped for getting the edible starchy kernel out. The popped kernel is known as *makhana* in India.

The processing of gorgon nut consists of conditioning, roasting at high temperature and popping of nut. Conditioning creates a small gap between the kernel and the shell as well as gelatinizes the kernel's starch. High temperature roasting generates super-heated steam which possibly causes building-up of pressure within the nut. This internal pressure and thermal treatment causes expansion of nut during roasting. Longer period roasting increases the volume of nut to an extent that it starts bursting which causes injuries to processors besides resulting unacceptable quality of *makhana*. Before self-bursting of nut during roasting, it has to be hit by the wooden mallet to get the uniformity in size, regularity in shape and good expansion ratio. These characteristics of *makhana* depend on magnitude of internal pressure and, thus, change in volume of the nut during roasting. The knowledge of magnitude and trend of change in volume of nut during roasting may help in controlling the popping process and optimization of its parameters for gorgon nut processing.

The objectives of the present work are to develop a mathematical model for prediction of change in volume of gorgon nut during roasting, and to examine the accuracy of predicted values experimentally.

MATHEMATICAL MODELLING

A conditioned gorgon nut comprised an outer shell, the starchy kernel and a small gap between the kernel and the shell filled with air-water vapour mixture (Fig 1). The change in volume of nut due to thermal treatment mainly involves three problems inter-linked due to the volume constraint as:

(i) simultaneous expansion and shrinkage of the kernel due to heating and moisture loss, respectively during roasting;

(ii) the compression of air-water vapour mixture in the gap between the kernel and the shell; and

(iii) thermo-elasticity of the spherical shell.

If V_s, V_k and V_a be the volumes enclosed by the shell, volume of the kernel and volume of the gap between the kernel and the shell respectively, then

$$V_s = V_k + V_a \quad (1)$$

For small change in volume

$$\Delta V_s = \Delta V_k + \Delta V_a \quad (2)$$

Change in Volume of the Kernel, ΔV_k

The change in volume of the kernel can be obtained by heat and mass transfer analysis of the kernel. For deducing the expressions for moisture transfer from the kernel and heat transfer to the kernel from the shell, following assumptions were made:

(i) the kernel is incompressible and spherical in shape,

(ii) heat transfer to the kernel from the shell is through conduction during roasting, and

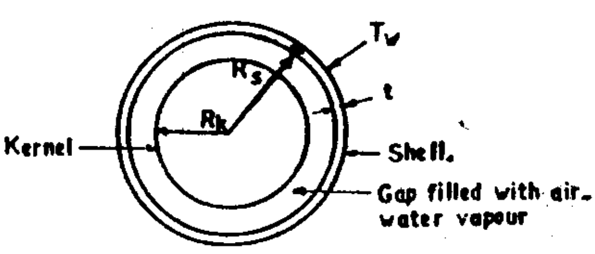

Fig 1 Spherical model of conditioned gorgon nut

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(iii) specific heat and latent heat of vaporisation of kernel are constant at a particular roasting temperature.

Moisture Transfer

The moisture content of kernel during roasting is dependent on initial and equilibrium moisture content of whole nut, roasting time and temperature. The relationship between the moisture content of kernel and the nut with roasting temperature and time can be defined by the following functions:

$$M_k = f(M_n) \quad (3)$$

$$M_n = f_1(D_n, \theta) \quad (4)$$

$$D_n = f_2(M_{no}, T, M_{ne}) \quad (5)$$

where, M_k, M_n, D_n, θ, M_{no}, T, and M_{ne} respectively are moisture content of kernel, decimal, db; moisture content of nut, decimal, db; moisture diffusivity of nut, $m^2\ min^{-1}$; roasting time, min; initial moisture content of nut, fraction, db; roasting pan temperature, °C; and equilibrium moisture content of nut, decimal, db.

Substituting equations (4) and (5) into equation (3), the final functions for M_k can be written as:

$$M_k = f[f_1\{f_2(M_{no}, T, M_{ne})\}, \theta] \quad (6)$$

The expressions for D_n, M_n and M_k assuming the equilibrium moisture content of the nut at high temperature conduction roasting as zero[2] are:

$$D_n = D_o + D_1 M_{no} + D_2 T + D_3 M_{no}^2 + D_4 M_{no} T \quad (7)$$

$$M_n = M_{no} C_1 \exp(-C_2 \theta) \quad (8)$$

$$M_k = C_3 M_n - C_4 \quad (9)$$

in which

$$C_2 = D_n \pi^2 / R^2$$

where, $D_0, D_1, D_2, D_3, D_4, C_1, C_2, C_3, C_4$ are constants and R is the radius of nut, m.

Putting the expression for M_n from equation (8) into equation (9)

$$M_k = C_1 C_3 M_{no} \exp(-C_2 \theta) - C_4 \quad (10)$$

The rate of moisture transfer from the kernel could be written as:

$$dM_k/d\theta = -C_1 C_2 C_3 M_{no} \exp(-C_2 \theta) \quad (11)$$

Heat Transfer

Heat input to the kernel from the shell by conduction (neglecting the convection and radiation loss) and heat transfer through the point contact, can be written as:

$$q_{in} = 4\pi k_v R_g R_k (T_w - T_k)/(R_g - R_k) \quad (12)$$

where, q_{in}, k_v, R_g, R_k, T_w, and T_k, respectively are heat input to the kernel, kW; thermal conductivity of air-water vapour mixture, $kWm^{-1}\ °C^{-1}$; radius of kernel plus gap between the shell and kernel, m; radius of kernel, m; shell wall temperature, °C; and kernel temperature, °C at any time.

The heat accumulated in the kernel

$$q_{ac} = C_{pk} W_{kd} (1 + M_k)(dT_k/d\theta) \quad (13)$$

where, q_{ac}, C_{pk}, W_{kd} are heat accumulated in the kernel, kW; specific heat of kernel, $kJ\ kg^{-1}\ °C^{-1}$; and bone dried mass of kernel, kg; respectively.

The heat utilized for vaporisation of moisture

$$q_{out} = -L_k W_{kd} (dM_k/d\theta) \quad (14)$$

where, q_{out} and L_k are heat utilized in vaporization of moisture of kernel, kW and latent heat of vaporization of moisture of the same, $kJkg^{-1}$, respectively.

Now,

$$q_{ac} = q_{in} - q_{out} \quad (15)$$

On substituting the equations (12), (13) and (14) into equation (15)

and rearranging the terms

$$dT_k/d\theta = 4\pi k_v R_g R_k (T_w - T_k)/\{C_{pk} W_k (R_g - R_k)\} + [L_k/\{C_{pk}(1 + M_k)\}](dM_k/d\theta)$$

Let $W_{sd}/W_{nd} = \delta_1$ then

$$W_k = W_{nd}[1 + M_n - \delta_1(1 + M_s)]$$

where, W_{sd}, W_{nd}, W_k and M_s are bone dried mass of shell, kg; bone dried mass of whole nut, kg; mass of kernel, kg; and moisture content of shell, fraction, db; respectively.

Replacing the mass of the kernel in terms of mass of nut and rearranging the terms in the above equation, the simplified form of heat transfer equation can be written as:

$$dT_k/d\theta = A(T_w - T_k) + B(dM_k/d\theta) \quad (16)$$

where,

$$A = 4\pi k_v R_g R_k (1 + M_{no})/[C_{pk} W_{nd}\{(1 + M_n) - \delta_1(1 + M_s)\} \times (R_g - R_k)], \text{ and}$$

$$B = L_k/\{C_{pk}(1 + M_k)\}$$

If $T_w - T_k = T$, $dT/d\theta = -dT_k/d_\theta$ and putting the expression for $dM_k/d\theta$ from equation (11), equation (16) can be written as:

$$-dT/d\theta = AT - BC_1C_2C_3 M_{no} \exp(-C_2\theta) \quad (17)$$

Let, $B_1 = BC_1C_2C_3 M_{no}$ and

$$C = B_1 \exp(-C_2\theta)$$

The equation (17) can be written in simpler form as:

$$dT/d\theta + AT = C \quad (18)$$

The equation (18) is first order differential equation which could be solved[3] as:

$$T = D\, e^{-A\theta} + B_1\, e^{-C_2\theta}/(A - C_2) \quad (19)$$

The value of constant D could be evaluated by putting initial condition at $\theta = 0, T = T_{ko}$ in equation (19). On rearranging the terms, the final equation for T_k is as follows:

$$T_k = T_w - B_1 [\exp(-C_2 \theta) - \exp(-A\theta)]/(A - C_2) - [(T_w - T_{ko}) \exp(-A\theta)] \quad (20)$$

From equation (10) and (20), the moisture content and temperature of the kernal could be obtained respectively at different time interval during roasting.

Now change in volume of the kernel due to change in moisture and temperature can be written as:

$$\Delta V_k = V_k [\gamma (T_k - T_{ko}) - \gamma_c (M_{ko} - M_k)] \quad (21)$$

where, γ and γ_c are coefficient of volume thermal expansion, $°C^{-1}$ and coefficient of moisture contraction, dimensionless, respectively.

Change in Volume of Air-water Vapour Mixture, ΔV_a

Let, P_o, V_o, T_o and m be the initial pressure, volume, absolute temperature, and mass of the air-water vapour mixture in the gap under ambient conditions, respectively. Assuming the shell of nut impermeable to the air-water vapour mixture for the short period of roasting, the changes in pressure, volume, temperature and mass during roasting period of θ be ΔP, ΔV, ΔT and Δm, respectively.

If the air-water vapour mixture obeys the ideal gas law, it can be written for ambient condition[4] as:

$$P_o V_o = m G_o T_o \quad (22)$$

and after elapsed time θ

$$(P_o + \Delta P)(V_o + \Delta V) = (m + \Delta m) G_1 (T_o + \Delta T) \quad (23)$$

where, G_0 and G_1 are gas constants ($Jkg^{-1}\ °C^{-1}$) at initial conditions and after time θ, respectively.

Dividing equation (23) by equation (22)

$$T_o (P_o + \Delta P)(V_o + \Delta V)/\{(T_o + \Delta T) P_o V_o\} = G_1 (m + \Delta m)/G_o m$$

As $\Delta V = \Delta V_a$ and $V_o = V_a$

$$\Delta V_a = V_a [\{G_1 P_o (m + \Delta m)(T_o + \Delta T)\} / \{G_o m T_o (P_o + \Delta P)\} - 1] \quad (24)$$

Let the volume of the air-water vapour gap V_a be a fraction (δ) of the total enclosed initial volume of the shell, V_s

therefore,

$$V_a = \delta V_s \quad (25)$$

and

$$V_k = (1 - \delta) V_s \quad (26)$$

Now substituting the expressions for ΔV_k and ΔV_a from the equations (21) and (24), respectively in equation (2) as:

$$\Delta V_s = V_k [\gamma (T_k - T_{ko}) - \gamma_c (M_{ko} - M_k)] + V_a [\{G_1 P_o (m + \Delta m)(T_o + \Delta T)/G_o m T_o (P_o + \Delta P)\} - 1] \quad (27)$$

Replacing V_a and V_k by equations (25) and (26), respectively in the above equation and rearranging the terms,

$$\Delta V_s = V_s [(1 - \delta) \{\gamma (T_k - T_{ko}) - \gamma_c (M_{ko} - M_k)\} + \delta \{G_1 P_o (m + \Delta m)(T_o + \Delta T)\}/\{G_o m T_o (P_o + \Delta P)\} - 1] \quad (28)$$

The equation (28) contains two unknown variables, *ie*, ΔV_s and ΔP, thus solution for ΔV_s can be obtained if another equation with same unknowns is derived.

Change in Volume Enclosed by the Shell, ΔV_s

Change in volume of nut was assumed to be enclosed by the shell. The expression for ΔV_s can also be derived in terms of properties of the shell. Before deriving the expression for change in volume enclosed by the shell (ΔV_s), the following assumptions were made:

(i) the shell of the nut is spherical in shape and the shell material of the nut is elastically isotropic;

(ii) the thermal anisotropy of the shell material is restricted to the radial direction only;

(iii) thickness of the shell is small as compared to its radius, and there is no abrupt change in curvature, so that, bending stress can be neglected;

(iv) roasting pan temperature is very high and the moisture content of thin shell of the nut is so low that, it attains an equilibrium moisture content and temperature almost instantaneously; and

(v) the temperature of the shell wall remains uniform with respect to thickness, so there is no thermal stress within the shell.

The net radial expansion of a thin spherical shell subjected to an internal pressure ΔP (Fig 2), uniform temperature field T_w (Fig 1), and radial contraction due to moisture loss could be obtained by the membrane theory of shells of revolutions as follows:

Considering the above assumptions, the simpler 'membrane theory' for shell[5] may be written as:

$$\sigma_\theta = \sigma_\phi = \Delta P R_s / 2t \quad (29)$$

where, σ_θ, σ_ϕ, ΔP, R_s, and t, respectively are circumferential stress, Pa; meridonal stress, Pa; internal pressure, Pa; radius of shell, m; and thickness of shell of nut.

The circumferential membrane strain in the shell is given by

$$\varepsilon_\theta - \varepsilon_{th} + \varepsilon_m = (\sigma_\theta - \nu \sigma_\theta)/E \quad (30)$$

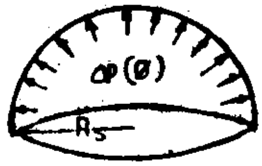

Fig 2 Force diagram for a spherical shell of gorgon nut under internal pressure

where, ε_θ, ε_{th}, ε_m, ν and E are elastic circumferential strain, thermal strain, circumferential moisture strain, Poisson's ratio, and modulus of elasticity (Pa) of shell of gorgon nut, respectively.

Thermal strain is given as:

$$\varepsilon_{th} = \alpha \Delta T_s \quad (31)$$

where, α and ΔT_s are coefficient of linear thermal expansion of shell, $^\circ C^{-1}$; and change in temperature of shell, °C, respectively.

Elastic circumferential strain is given by

$$\varepsilon_\theta = \Delta R_s / R_s \quad (32)$$

where, R_s and ΔR_s are radius of shell, m and change in the same, respectively.

Circumferential moisture strain is expressed as:

$$\varepsilon_m = \beta \Delta M_s \quad (33)$$

where, β and ΔM_s are the coefficient of linear moisture contraction (dimensionless) and change in moisture content, of shell, fraction, db, respectively.

From equation (30)

$$\varepsilon_\theta = (\sigma_\theta - \nu \sigma_\theta)/E + \varepsilon_{th} - \varepsilon_m \quad (34)$$

Substituting the equations (29), (31), (32), (33) into equation (34) yields:

$$\Delta R_s = R_s \left[(\Delta P R_s / 2tE)(1-\nu) + \alpha \Delta T_s - \beta \Delta M_s\right] \quad (35)$$

The change in volume enclosed by the shell becomes as:

$$\Delta V_s = 4\pi \left[(R_s + \Delta R_s)^3 - R_s^3\right]/3$$

Neglecting the terms containing the higher order of ΔR_s, the simplified form of above equation can be written as:

$$\Delta V_s \simeq 4\pi R_s^2 \Delta R_s \quad (36)$$

Putting the expression for ΔR_s from equation (35) to equation (36), the expression for change in volume enclosed by the shell can be obtained as:

$$\Delta V_s = 3V_s \left[(\Delta P R_s / 2tE)(1-\nu) + \alpha \Delta T_s - \beta \Delta M_s\right] \quad (37)$$

Now, equations (28) and (37) are two expressions for ΔV_s which could be solved simultaneously for change in volume of the nut during roasting.

MATERIALS AND METHODS

Solution of Mathematical Model

The developed model was solved by employing the technique of simultaneous solution of two equations having two unknowns. A computer program in FORTRAN-77 was developed and used on a mainframe computer Cyber 180/120. The material temperature of 213°C, when the nut was roasted at 335°C pan temperature for highest percentage of popping of gorgon nut[6], was taken for solution. The basic data required for solution of the model were determined separately and used in computer program (Table 1)[2,6].

Table 1 Properties of gorgon nut and other data used in prediction of change in volume of nut during roasting[2,6]

Radius of nut	4.5 mm
Shell thickness	1.0 mm
Gap between shell and kernel	0.2 mm
Poisson's ratio of nut	0.3
Modulus of elasticity	940×10^6 Pa
Coefficient of linear thermal expansion of nut	3.88×10^{-4}/°C
Coefficient of linear moisture contraction of nut	0.0152
Coefficient of volume thermal expansion of kernel	23.95×10^{-4}/°C
Coefficient of volume moisture contraction of kernel	0.0603
Initial moisture content of nut	25.9% (db)
Initial temperature of nut	30°C
Final temperature of nut	213°C
Initial mass of a single nut	4.585×10^{-4} kg
Initial moisture content of kernel	33.4% (db)
Initial moisture content of shell	5% (db)
Final moisture content of shell	2%
Latent heat of vaporisation of kernel moisture	2175.7 kJ/kg
Specific heat of kernel	2.152 kJ/kg °C
Thermal conductivity of vapour	1.41 J/m-min-°C
Ratio δ_1	0.513
Normal atmospheric pressure	101.3 kPa
Normal temperature	303.15 K
C_1	0.979
C_3	1.357
C_4	0.75

Measurement of Change in Volume of Nut

To measure the change in volume of the nut during roasting, 10 medium size sound conditioned nuts were randomly selected for one replication and roasted one by one in a cast iron pan at 335°C surface temperature for five minutes. Conditioning of nut was carried out as per procedure[7]. The diameter of the nut was measured at 0.5 min interval using a vernier caliper having the least count of 0.001 mm. The change in volume with respect to initial one was computed. The experiment was replicated five times and the average values were reported.

RESULTS AND DISCUSSION

The predicted change in volume of nut during roasting increased rapidly in the beginning, and later its rate of increase declined (Table 2). This may be due to higher rate of moisture migration and temperature rise at initial stage and accumulation of the former in the gap between the shell and the kernel which may build-up pressure within the nut rapidly. The predicted change in volume at 2.5 min of roasting time and 213°C nut temperature was found to be 142.1 mm³ which is 37.2% of initial volume of the nut. The high rate of increase in volume reveals that the nut may burst on its own during

Table 2 Comparison of predicted and measured values of change in volume of nut at 335°C roasting pan temperature

Roasting Time, min	Change in Volume of Nut, mm^3					Variation from Measured Values, %	
	Measured			Predicted			
	Absolute		Per cent of Initial	Absolute	Per cent of Initial	Absolute	Per cent of Initial
0.0	0.0	—	—	0.0	—	—	—
0.5	15.1	(1.7)*	3.9	98.72	25.9	554.2	564.1
1.0	38.8	(2.3)	10.2	117.54	30.8	202.9	202.0
1.5	80.7	(4.5)	21.1	129.07	33.8	60.0	60.2
2.0	121.5	(4.1)	31.7	136.60	35.8	12.4	12.9
2.5	129.5	(3.2)	33.9	142.10	37.2	9.7	9.7
3.0	134.3	(4.6)	35.2	146.51	38.4	9.1	9.1
3.5	139.6	(4.2)	36.5	150.22	39.4	7.6	7.9
4.0	143.6	(4.5)	37.6	153.42	40.2	6.8	6.9

*Figures in parenthesis are standard deviations.

roasting which may cause injuries to the nearby persons and inferior quality of *makhana*. The experiments conducted to measure the change in volume of nut showed bursting of few nuts after about 2.5 min of roasting and yielded fractured *makhana*. It suggested that nut should be hit externally before 2.5 min to avoid the self-bursting of nut for having good quality of *makhana*. This result is also with the agreement[8] of the optimum roasting time, 2.5 ± 0.2 min.

The predicted and measured values of changes in volume of the nut, as far as examination of validity of model is concerned, were compared (Table 2). It may be noted from the table that variation between predicted and measured values was very high(60% to 554.2%) till 1.5 min of roasting. The large variation in the initial stage may be due to the assumption made in formulating the model that the shell of the nut attains an equilibrium temperature almost instantaneously at high temperature conduction roasting which may not be completely valid in real situation. However, the variation between the observed and predicted values after 2 min was within 10% which suggests that the model could be used well for the purpose beyond 2 min of roasting time.

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